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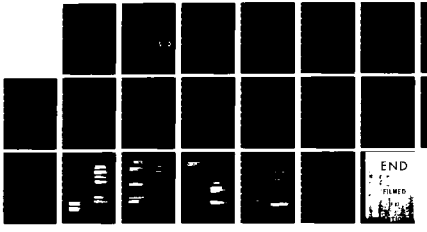

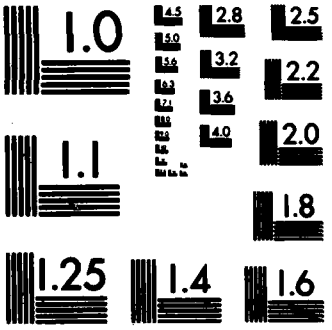

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(1) The methods of conformal mapping and analytic continuation are applied to obtain slender-body centerplane distributions,

(2) Three closed-form approximations to the double-integral "near-field" term in the Havelock Green function are derived. An application to centerplane source distributions for ship-wave problems is discussed.

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VISCOUS EFFECT ON SHIP WAVE RESISTANCE

FINAL REPORT

Sponsored by

General Hydromechanics Research Program
of the Navy Sea Systems Command
David W. Taylor Naval Ship Research & Development Center
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Iowa Institute of Hydraulic Research
The University of Iowa
Iowa City, Iowa 52242

July 1983

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Final Report

VISCOUS EFFECT ON SHIP WAVE RESISTANCE

Work on the subject problem under the sponsorship of DWTNSRDC was initiated in October 1981 and terminated in September 1982. This research is being continued under the Special Focus Program of the Office of Naval Research.

During the contract year, one paper was published and two reports were produced. These are the following:

1. "Centerline Distribution as Solutions of Neumann Problems for Ogivelike Forms", by L. Landweber, Eighteenth Midwestern Mechanics Conference, May 1983.

2. Progress Reports by Kazuhiro Mori.
   A. Calculation of Wave Resistance and Sinkage by Rankine-Source Method.
   B. Prediction of 2-D Near Wake Flow by Making Use of Time-Dependent Vorticity Transport Equation.
   C. Free-Surface Boundary Layer and Necklace Vortex Formation.

   IIHR Report No. 262, The University of Iowa, May 1983.

3. "Numerical Evaluation of the Havelock Green Function", by Lawrence K. Forbes, unpublished. This is attached as part of this Final Report.

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CENTERLINE DISTRIBUTIONS AS SOLUTIONS OF NEUMANN PROBLEMS FOR OGIVELIKE FORMS

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Introduction

As in well known, approximate solutions of symmetric Neumann problems for thin, symmetric two-dimensional forms can be obtained by formulating the boundary condition as a Fredholm integral equation of the first kind. Since exact solutions of such integral equations exist only when certain conditions are satisfied, it is of interest to find cases for which exact solutions can be obtained.

The current interest in this problem stems from an attempt to find a centerplane distribution for a slender ship form with ogive-like sections. Slender-body theory would yield an approximate centerplane distribution for such a form if the particular Neumann problems posed by the slender-body theory have exact solutions for the transverse sections of the double ship form.

In the present work, we shall be mainly concerned with solutions for ogival sections. Applications of the procedures to other forms, such as elliptical and doubly-parabolic (Wigley) sections will be indicated.

Geometry

The ogive is a two-dimensional form derived from the intersection of two circular arcs of equal curvature. For a form of thickness 2c and length 2h, its equation is

$$|y| = \frac{1}{2c}\left\{\left[(h^2 + c^2)^2 - 4c^2x^2\right]^{1/2} - h^2 + c^2\right\}, \quad -h \leq x \leq h \qquad (1)$$

its slope at $x = -h$ is

$$\tan \alpha = \frac{2ch}{h^2 - c^2} , \quad c = h \tan \frac{\alpha}{2} \tag{2}$$

and its radius of curvature is $R = h \sec \alpha$. The cosine of the angle of the normal to the ogive with the x- axis is

$$\frac{\partial x}{\partial n} = \frac{x}{h} \sin \alpha \tag{3}$$

The exterior of the ogive in the $z = x + iy$ plane is mapped into the exterior of the unit circle in the plane $\zeta = \xi + i\eta = \rho e^{i\phi}$ by the successive transformations

$$Z = \frac{z - h}{z + h} , \quad T = Z^{\lambda} , \quad T = \frac{\zeta - 1}{\zeta + 1} \tag{4}$$

where

$$\lambda = \frac{\pi/2}{\pi - \alpha} , \quad 1/2 < \lambda < 1 \tag{5}$$

Mappings of the ogive in the z, Z, T and ζ- planes are shown in Fig. 1. The upper half of the ogive maps into the crescent ACBDA in the ζ- plane; the lower half (not shown) would give the mirror image of the crescent.

Let θ_2, ϕ, ϕ_2 denote the polar angles in the Z, ζ, and T- planes, and put

$$\omega = (\tan \phi/2)^{1/\lambda} , \quad E = [\omega^2 + 2\omega \cos \alpha + 1]^{1/2} \tag{6}$$

Then $\phi_2 = \lambda \theta_2$ and the locations of points in the mappings of Fig. 1 are given in the following table:

	P	D	C	E
z- pl	∞	$i \tan \alpha$	0	$-i \cot \alpha$
Z- pl	1	$-e^{i\alpha}$	-1	$e^{-2i\alpha}$
T- pl	1	i	$ie^{i\beta}$	-1
ζ-pl	∞	i	$i \tan (\frac{\pi}{4} - \frac{\beta}{2})$	0

$$\beta = \frac{\alpha}{2}/(1 - \frac{\alpha}{\pi})$$

Solutions of the Neumann Problem

We wish to find solutions of the Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{7}$$

for the exterior of the ogive which have zero circulation and, on its contour, satisfy the boundary condition

$$\frac{\partial \phi}{\partial n} = f(x) \tag{8}$$

where n denotes distance normal to the ogive, positive outwards. The form of the Neumann boundary condition (8) implies that $\partial\phi/\partial n$ is the same at $\pm\, y(x)$. The flux through the ogive is given by

$$Q = 2\pi M = 2 \int_{-h}^{h} f(x) \frac{ds}{dx}\, dx \tag{9}$$

where s denotes arc length along the ogive, positive in the counterclockwise sense.

In the ζ- plane, the Neumann boundary condition becomes

$$\frac{\partial\Phi}{\partial\rho} = f(x) \left|\frac{dz}{d\zeta}\right| \,, \quad \rho = 1 \tag{10}$$

For the ogive, we have

$$x = h\,(1 - \omega^2)/\,E^2 \tag{11}$$

and

$$\left|\frac{dz}{d\zeta}\right| = \frac{2\,h\,\omega}{\lambda\,E^2 \sin\phi} \tag{12}$$

We define a zero-flux potential Φ_0 by

$$\Phi = \Phi_0 + M \ln\rho\,, \quad \frac{\partial\Phi_0}{\partial\rho} = \frac{\partial\Phi}{\partial\rho} - \frac{M}{\rho} \tag{13}$$

The complex potential for this Neumann problem is then

$$w(\zeta) = \Phi + i\psi = M \ln\zeta + \frac{1}{\pi} \int_0^{2\pi} \frac{\partial\Phi}{\partial\rho'} \ln\left(\zeta - e^{i\phi'}\right) d\phi', \quad \rho > 1 \tag{14}$$

and the complex velocity is

$$\frac{dw}{d\zeta} = \frac{M}{\zeta} + \frac{1}{\pi} \int_0^{2\pi} \frac{\partial\Phi}{\partial\rho'} \frac{d\phi'}{\zeta - e^{i\phi'}}, \quad \rho > 1 \tag{15}$$

The form of $dw/d\zeta$ in (15) cannot be used to continue the complex velocity analytically into the unit circle. If none of the singularities of $dw/d\zeta$ lies within the crescent ACBDA in Fig. 1, the function can be continued analytically to the arc BCA in the ζ- plane, and hence to the axis of the ogive in the z- plane. We would then have, at the ogive axis,

$$\frac{dw}{dz} = \frac{dw}{d\zeta}\frac{d\zeta}{dz} = u\,(x,0) - iv\,(x,\,0^+) \tag{16}$$

where u and v are velocity components in the x- and y- directions, and 0^+ indicates that $y \to 0$ through positive values. The axial source distribution would then be given by

$$m(x) = \frac{1}{\pi} v(x,0) = -\frac{1}{\pi} \mathrm{Im} \frac{dw}{dz} \tag{17}$$

where Im denotes the imaginary part.

Example: Ogive in a Uniform Stream

For this case, the exact centerline distribution can be found by a simpler method and is given by

$$m(x) = \frac{16}{\pi} \lambda^2 h^2 \sin 2\pi\lambda \frac{[(h+x)^{4\lambda} - (h-x)^{4\lambda}](h^2 - x^2)^{2\lambda - 1}}{[(h+x)^{4\lambda} - 2(h^2 - x^2)^{2\lambda} \cos 2\pi\lambda + (h-x)^{4\lambda}]^2} \tag{18}$$

More generally, however, an explicit algebraic formula for the complex velocity, as was used to derive (18), is not available. An alternative procedure is to investigate the singularities of the integrand of (15), expressed as a line integral around the unit circle of a function of $\zeta' = e^{i\phi'}$. If $\partial\phi/\partial\rho'$ can be expressed as a function $F(\zeta')$, (15) could be written as

$$\frac{dw}{dz} = \frac{M}{\zeta} - \frac{i}{\pi} \oint F(\zeta') \frac{d\zeta'}{\zeta'(\zeta - \zeta')} \tag{19}$$

In the present case, $M = 0$ and, by (3)

$$f(x) = \frac{\partial\phi}{\partial n} = -\frac{\partial x}{\partial n} = -\frac{x}{h} \sin\alpha \tag{20}$$

and, by (10), (11) and (12),

$$\frac{\partial\phi}{\partial\rho'} = -\frac{2h\omega(1 - \omega^2)\sin\alpha}{\lambda E^4 \sin\phi'}, \quad \rho' = 1 \tag{21}$$

Since $\sin\phi' = (\zeta'^2 - 1)/2i\zeta$, and one can show that

$$\omega = -\left(\frac{\zeta' - 1}{\zeta' + 1}\right)^{1/\lambda} e^{i\alpha} = -Z e^{i\alpha} \tag{22}$$

we see that $\partial\phi/\partial\rho'$ can be expressed as $F(\zeta')$, and (19) becomes

$$\frac{dw}{d\zeta} = -\frac{4h \sin\alpha}{\pi\lambda} \oint \frac{\omega(1 - \omega^2)\, d\zeta'}{E^4 (\zeta'^2 - 1)(\zeta - \zeta')}, \quad \rho > 1 \tag{23}$$

The apparent singularity at $\zeta' = \pm 1$ is removable. The partial-fraction form

$$\frac{\omega\,(1-\omega^2)}{\xi^4} = \frac{i}{2\sin\alpha}\left[\frac{1}{\omega e^{-i\alpha}+1} - \frac{1}{(\omega e^{-i\alpha}+1)^2} - \frac{1}{\omega e^{i\alpha}+1} + \frac{1}{(\omega e^{i\alpha}+1)^2}\right] \qquad (24)$$

then shows where poles of the integrand may be present. We find that $(\omega e^{-i\alpha}+1)$ has no zeros within the unit circle, but that

$$\omega e^{i\alpha} + 1 = 2\,(\mu\zeta' - \mu^2\zeta'^2 + \ldots) \qquad (25)$$

vanishes at $\zeta' = 0$. Hence, since there are no singularities within the crescent, this confirms the existence of an exact centerline distribution.

It is also of interest to attempt to evaluate the integral in (23). In order to apply the Cauchy residue theorem, it is necessary that the integrand be a regular function, except at $\zeta' = 0$. Because of symmetry, the imaginary parts of the integrands at conjugate points in the unit circle must have opposite signs when $\zeta = \xi > 1$. These two requirements can be satisfied only if the integrand is real along the diameter $\zeta' = \xi'$, $-1 < \xi' < 1$. It can be shown that this last condition is satisfied by the third and fourth terms of (24). Since these are also the terms which, by (25), yield the singularity, one may apply the residue theorem to that part of the integrand containing these two terms. By applying (25), we obtain

$$\frac{1}{(\zeta-\zeta')(\zeta'^2-1)}\left[\frac{1}{(\omega e^{i\alpha}+1)^2} - \frac{1}{\omega e^{i\alpha}+1}\right] = -\frac{1}{4\mu^2\zeta^2\zeta'} + \ldots.$$

The expression (23) for the complex velocity then becomes

$$\frac{dw}{d\zeta} = -\frac{2ih}{\pi\lambda}\oint\left[\frac{1}{\omega e^{-i\alpha}+1} - \frac{1}{(\omega e^{-i\alpha}+1)^2}\right]\frac{d\zeta'}{(\zeta'^2-1)(\zeta-\zeta')} - \frac{\lambda h}{\zeta^2} \qquad (26)$$

Comparison with the known exact expression

$$\frac{dw}{d\zeta} = \lambda h\left(1 - \frac{1}{\zeta^2}\right) - \frac{dz}{d\zeta} \qquad (27)$$

indicates the value of the remaining integral in (26). Since the present example was treated for the purpose of illustrating a procedure for finding centerline distributions, and, in general, integrals such as that in (26), would require numerical evaluation, further consideration of this case would not be rewarding.

Procedure for Ogivelike Sections

1. Map the given section in the z- plane into the unit circle in the ζ- plane. For an ogivelike section, this is best done with the preliminary transformation (4) to transform the section first into a nearly circular form.

2. Map the axis of the ogive into the interior of the unit circle. The crescentlike region bounded by this curve and the arc of the unit circle gives the mapping of the upper (or lower) half of the given section.

3. Obtain $\partial\Phi/\partial\rho$, the Neumann boundary condition on the unit circle, express it as function $F(\zeta)$, $\zeta = e^{i\phi}$, and determine the singularities of the integral expression (15) for $dw/d\zeta$. If none of the singularities lies within the crescentlike region, then there exists a centerline distribution of sources which gives the solution of the given Neumann problem for the section.

4. If the solution exists, it can be obtained by solving numerically a Fredholm integral equation of the first kind formulated by equating the normal component at the surface of the given profile, induced by the unknown axial source distribution, to the prescribed Neumann boundary condition.

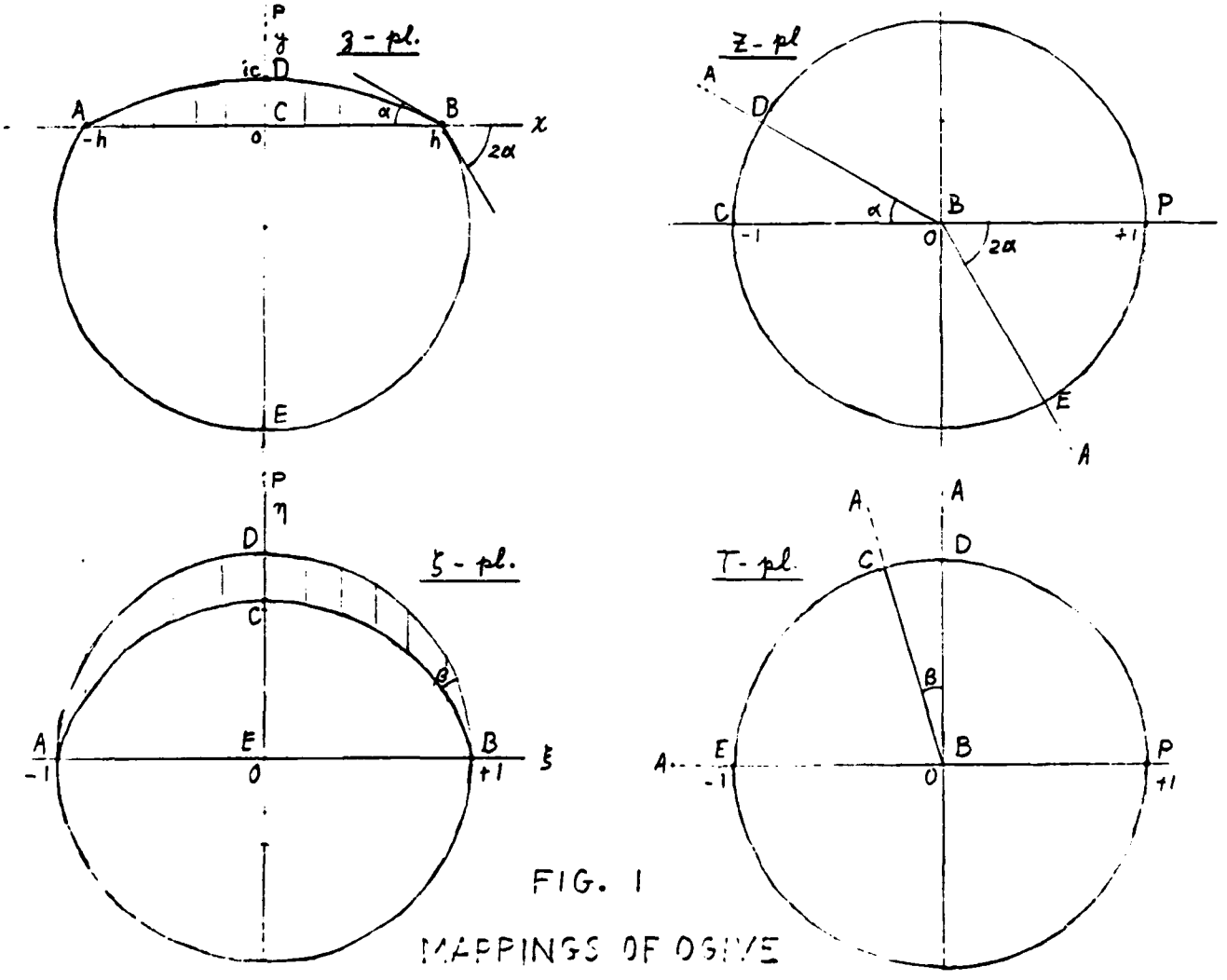

FIG. 1

MAPPINGS OF OGIVE

NUMERICAL EVALUATION OF THE HAVELOCK GREEN FUNCTION

by

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Abstract

Three closed-form approximations to the double-integral "near-field" term in the Havelock Green function are derived. An application to centerplane source distributions for ship-wave problems is discussed.

1. Introduction

In the theory of potential flow about arbitrary bodies moving steadily in or beneath a linearized free surface, the velocity potential is usually sought as the solution to an integral equation having as the kernel a derivative of the Havelock Green function in the direction normal to the body surface. The choice of the Havelock Green function as the fundamental solution to Laplace's equation is natural, since it immediately ensures that the linearized free-surface condition and the radiation condition are satisfied. However, this important advantage is offset by the great complexity of the function itself, forcing some investigators [1, 2, 3] to abandon it completely in favor of the simpler Rankine source function.

Because of the central role played by the Havelock function in the theory of ship wave resistance, its numerical evaluation has been the subject of much research. There are many different expressions for this function, all of which may be related to one of the three basic forms summarized by Noblesse [4], who simplified the form of the function somewhat through the introduction of the complex exponential integral.

In the numerical application of the Havelock Green function to the solution of practical ship-wave problems, there are two basic difficulties to overcome. The first of these is due to the additional singular behavior of this Green function on the plane of the linearized free surface, described by Noblesse [4]. This problem may be circumvented by adopting the approach of Miloh and Landweber [5]. They showed that, under certain assumptions about the ship-hull geometry, the problem could be reduced to finding an appropriate source distribution M on the ship centerplane satisfying the first-kind Fredholm integral equation

$$\int M(P) \frac{\partial G(Q,P)}{\partial N_Q} dS_P + \frac{\partial x}{\partial N_Q} = 0 \qquad (1)$$

on the ship hull

$$y = \pm f(x,z) , \qquad (2)$$

where $P(\xi,0,\zeta)$ and $Q(x,y,z)$ are points on the centerplane and ship hull, respectively. The Havelock Green function is denoted by the symbol $G(Q,P)$ and N_Q represents the direction normal to the hull. By discretizing the integral on the left-hand side of equation (1) using Gaussian quadrature, the requirement that numerical grid points be placed on the plane $z = 0$ of the undisturbed surface is eliminated, and the singular nature of the Havelock Green function on this plane is avoided. The results of the present paper have been developed primarily with a view to their eventual implementation in the numerical solution of equation (1).

The other major difficulty with the Havelock Green function is the computing time required to evaluate it. On the ship hull, in particular, the "near-field" component of this Green function, which may be expressed in terms of a double integral, cannot reasonably be ignored and an efficient method for its evaluation must be developed. A simple calculation serves to illustrate the point; if a solution to equation (1) is sought, employing 50 points from bow to stern and 20 points from keel to free surface in the discretization of the integral (a total of 1000 points), then 10^6 separate evaluations of the Green function would be required to form the kernel of the integral equation. A single accurate evaluation typically requires in excess of 1/10 th sec. computing time, so that approximately 10^5 secs. or <u>one day</u>

continuous computing time would be required to solve the problem, which is unreasonable. Such considerations lead Lee [6] to conclude that the function cannot be applied to the solution of practical flow problems.

In the present paper, a number of different approximations to the basic function ([7], page 484)

$$G(Q,P) = -\frac{1}{R} - \frac{1}{R'} + \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty} \frac{\exp\left[kz' - ik\rho\cos(\theta - \alpha)\right]}{k - \dfrac{1}{F^2\cos^2\theta}}\, k\, dk\, d\theta$$

$$- 2\int_{-\pi/2}^{\pi/2} \frac{\exp\left[\dfrac{z'}{F^2\cos^2\theta}\right]\sin\left[\dfrac{\rho\cos(\theta - \alpha)}{F^2\cos^2\theta}\right]}{F^2\cos^2\theta}\, d\theta \qquad (3)$$

and its derivative in the direction normal to the hull (2) will be derived, and their suitability for the rapid evaluation of these functions determined. Here, F is a Froude number based on the ship length and R is the distance

$$R = \left[x'^2 + y'^2 + (z - \zeta)^2\right]^{1/2} \qquad (4)$$

between the points $Q(x, y, z)$ and $P(\xi, \eta, \zeta)$. The quantities x', y', z' are defined by the relations

$$\begin{aligned} x' &= x - \xi \\ y' &= y - \eta \\ z' &= z + \zeta. \end{aligned} \qquad (5)$$

In addition,

$$R' = [x'^2 + y'^2 + z'^2]^{1/2}, \tag{6}$$

and

$$\rho = [x'^2 + y'^2]^{1/2} \tag{7}$$

$$\alpha = \arctan (y'/x')$$

2. Direct Numerical Evaluation

In order to evaluate the Havelock Green function numerically, the Cauchy principal-valued integral appearing in equation (3) is first transformed by a straightforward contour integration due to Havelock ([8], page 291). After some algebra, equation (3) becomes

$$G(Q,P) = -\frac{1}{R} - \frac{1}{R'}$$

$$+\frac{2}{\pi}\int_{-\pi/2}^{\pi/2}\int_{0}^{\infty}\frac{k\, e^{-k\rho\cos\theta}}{k^2 + \dfrac{1}{F^4\cos^4(\theta+\alpha)}}\left[\frac{\cos kz'}{F^2\cos^2(\theta+\alpha)} - k\sin kz'\right] dk\, d\theta$$

$$-\frac{4}{F^2}\int_{-\pi/2+\alpha}^{\pi/2}\exp\left[\frac{z'}{F^2\cos^2\theta}\right]\sin\left[\frac{\rho\cos(\theta-\alpha)}{F^2\cos^2\theta}\right]\frac{d\theta}{\cos^2\theta}, \tag{8}$$

where the functions R, R', etc. are defined by equations (4) - (7).

The (indefinite) integral with respect to k in equation (8) may be evaluated using the Laguerre polynomial approach (see [9], page 890, formula 25.4.45). To demonstrate that acceptable accuracy could be maintained by the Laguerre polynomial method when the integrand is oscillatory, the test functions

$$\int_0^\infty e^{-x} \begin{Bmatrix} \cos x \\ \sin x \end{Bmatrix} dx$$

were evaluated using the fifteenth-order integration formula, and it was found that the result agreed with the exact answer 1/2 to six figures in both cases.

Once the k-integration in the double-integral term in equation (8) has been performed, it remains to integrate the result with respect to θ. This may be achieved using Gaussian quadrature. Similarly, the single-integral term in equation (8) may be evaluated by Gaussian integration.

Finally, the normal derivative

$$\frac{\partial G\ (Q,P)}{\partial N_Q} = \nabla_Q G(Q,P) \cdot \mathbf{N} \tag{9}$$

is usually required, as in equation (1), for example. Here,

$$\mathbf{N} = \frac{-f_x\,\mathbf{i} + \mathbf{j} - f_z\,\mathbf{k}}{[1 + f_x^2 + f_z^2]^{1/2}} \tag{10}$$

is the normal vector to the ship hull. The derivatives of G in the x -, y - and z-directions are computed using forward differences.

3. Asymptotic Expansion for Small Froude Number

To develop an asymptotic expansion for small Froude number, equation (3) is first expressed in the form

$$G = -\frac{1}{R} - \frac{1}{R'} + N_3 + W_3 \ ,$$

where

$$N_3 = \frac{2}{R'} + \frac{2}{\pi F^2} \text{Re} \int_{-\infty}^{\infty} e^{Z_3} E_1 (Z_3) \, dt$$

and

$$W_3 = -\frac{4}{F^2} \text{Im} \int_{-\pi/2 + \alpha}^{\pi/2} \sec^2\theta \exp \left[\frac{z' \sec^2 \theta + i\rho \sec^2 \theta \cos (\theta - \alpha)}{F^2}\right] d\theta \quad (11)$$

The complex number Z_3 in equation (11) is defined as

$$Z_3 = \frac{1}{F^2} \left[z' (1 + t^2) + i (x' + ty')(1 + t^2)^{1/2}\right] ,$$

and $E_1 (Z_3)$ is the complex exponential integral

$$E_1 (Z_3) = \int_{Z_3}^{\infty} \frac{e^{-\lambda}}{\lambda} d\lambda \, .$$

Equation (11) is the third of the forms of the Havelock Green function derived by Noblesse [4].

Since the evaluation of the exponential integral of the complex argument Z_3 is apparently more time consuming than the corresponding Laguerre-polynomial integration in section 2, equation (11) does not appear to be of much value in the direct numerical evaluation of the Havelock Green function, despite the fact that the double-integral term has been reduced to a single integral involving the exponential integral, which in some sense may be regarded as an "elementary" function. However, equation (11) readily yields an asymptotic expansion for low Froude number.

Since $|Z_3|$ becomes large as $F \to 0$, provided that x', y', and z' do not all simultaneously vanish, the function $E_1(Z_3)$ may be replaced by its asymptotic expansion for large $|Z_3|$,

$$E_1(Z_3) \sim e^{-Z_3}\left[\frac{1}{Z_3} + \sum_{n=1}^{\infty} \frac{(-1)^n\, n!}{Z_3^{n+1}}\right] .$$

Substituting this result into the formula for the near-field disturbance term N_3 in equation (11) yields the asymptotic expansion

$$N_3 \sim \frac{2}{R'} + \frac{2}{\pi F^2}\,\mathrm{Re}\left[\int_{-\infty}^{\infty} \frac{dt}{Z_3} + \sum_{n=1}^{\infty} (-1)^{n}\, n! \int_{-\infty}^{\infty} \frac{dt}{Z_3^{n+1}}\right] . \tag{12}$$

Each of the integrals in equation (12) may be evaluated in closed form, using the calculus of residues, but with greatly increased labor for every increase in the value of n.

We have computed the first three terms in the asymptotic expansion (12) for N_3, and present the results below. The first integral cancels with the first term on the right hand side of equation (12); then, after substantial calculation, the desired expansion is obtained in the form

$$\begin{aligned} N_3 \sim\ & 2F^2\left[\frac{x'^2 - y'^2}{\rho^4} + \frac{z'(x'^2 - y'^2)}{R'\rho^4} + \frac{x'^2 z'}{R'^3 \rho^2}\right] \\ & - 6F^4\left[\frac{x'^8 z'^6}{R'^5 r^4 \rho^8} + \frac{x'^6 z'^4 (5y'^2 + z'^2)}{R'^3 r^4 \rho^8}\right. \\ & + \frac{x'^4 z'^2 (11y'^2 z'^2 + 2z'^4 + 15y'^4)}{R' r^4 \rho^8} + \frac{2z'(x'^4 - 6x'^2 y'^2 + y'^4)}{\rho^8} \\ & \left. - \frac{R' x'^2 y'^2 (21y'^2 z'^2 + 12z'^4 + 5y'^4)}{r^4 \rho^8} + \frac{R'^3 y'^4 (y'^2 + 2z'^2)}{r^4 \rho^8}\right] \\ & + O(F^6) , \end{aligned} \tag{13}$$

where

$$r = [y'^2 + z'^2]^{1/2}.$$

To derive an asymptotic expansion for the wave-like contribution W_3 to the Green function in equation (11) we note that, for $\rho \neq 0$, the parameter ρ/F^2 becomes large as $F \to 0$, and consequently W_3 may be approximated using the method of stationary phase. The result is classical, and appears in numerous texts and articles on ship waves (see, for example, Stoker [10] and Newman [11]). This gives

$$W_3 \sim -\frac{4}{F} \sum_{i=1}^{2} \left(\frac{\pi}{\rho |g''(\theta_i)|}\right)^{1/2} \sec^2\theta_i \exp\left[\frac{z' \sec^2\theta_i}{F^2}\right]\left\{\cos\left(\frac{\rho g(\theta_i)}{F^2}\right)+\sin\left(\frac{\rho g(\theta_i)}{F^2}\right)\right\} \quad (14a)$$

for $|\tan\alpha| < 2^{-3/2}$, where θ_1 and θ_2 are found from

$$\tan\theta_{1,2} = \frac{-1 \pm (1 - 8\tan^2\alpha)^{1/2}}{4\tan\alpha},$$

and $g(\theta_i) = \sec^2\theta_i \cos(\theta_i - \alpha)$, $i = 1, 2$.

When $|\tan\alpha| > 2^{-3/2}$,

$$W_3 \sim 0, \quad (14b)$$

and for $\tan\alpha = \pm 2^{-3/2}$,

$$W_3 \sim -2^{3/2}\,\Gamma\left(\tfrac{1}{3}\right)\left(\frac{3}{\rho F^4}\right)^{1/3} \exp\left[\frac{3z'}{2F^2}\right] \sin\left[\frac{\rho\sqrt{3}}{2F^2}\right]. \quad (14c)$$

The low Froude number form of the Havelock Green function is thus given by equations (11), (13) and (14). The normal derivative (10) is computed using forward differences.

4. Approximate Green function for centerplane source distribution on a thin ship

An alternative form of the Havelock Green function (3), which also involves the exponential integral function E_1, is given by the expression

$$G = -\frac{1}{R} + \frac{1}{R'} + N_1 + W_1 ,$$

where

$$N_1 = \frac{2}{F^2}\left[\frac{1}{\pi}\int_{-1}^{1} \mathrm{Im}\{e^{Z_1} E_1(Z_1) + \ln Z_1 + \gamma\}\, dt - \left(1 - \frac{z'}{|x'| + R'}\right)\right]$$

and

$$W_1 = 4H(x')\int_{-\infty}^{\infty} \exp\left[\frac{(1 + F^4 t^2)\, z'}{F^2}\right] \sin\left[\left(-ty' - \frac{x'}{F^2}\right)(1 + F^4 t^2)^{1/2}\right] dt. \quad (15)$$

The complex number Z_1 is defined as

$$Z_1 = \frac{(1 - t^2)^{1/2}}{F^2}\left[i|x'| - ty' + (1 - t^2)^{1/2} z'\right] ,$$

and $H(x')$ is the Heaviside unit step function, having the value 0 when $x' < 0$ and 1 when $x' > 0$. The term $\ln Z_1 + \gamma$, where γ is Euler's constant, has been included to ensure that the integrand of the expression for N_1 is well behaved when $|Z_1| \to 0$.

Equation (15) is the first of the forms of the Havelock Green function derived by Noblesse [4], who recommends its use in the direct numerical evaluation of this function (see also Noblesse [12, 13]). Actually, equation

(8) appears to be better suited to this purpose, for the reason outlined in section 3. In addition, the use of equation (15) in an integral equation, such as equation (1), requires special treatment of the step discontinuity at $x' = 0$.

Equation (15) may be used to derive an approximation to the normal derivative of the Green function for centerplane source distributions. In this case, the point P lies on the centerplane $n = 0$, and the point Q is on the hull $y = f(x, z)$. The flow is assumed to be symmetrical about the plane $y = 0$, so that only one of the hull surfaces need be considered in equation (2). If $\partial N_1/\partial N$ denotes the normal derivative of the near-field term N_1 in equation (15), then

$$(1 + f_x^2 + f_z^2)^{1/2} \frac{\partial N_1}{\partial N}$$

$$= -\frac{2}{\pi F^4} \int_{-1}^{1} (1 - t^2)^{1/2} \operatorname{Im} \{e^{Z_1} E_1(Z_1)[i f_x \operatorname{sgn} x' + t + f_z(1 - t^2)^{1/2}]\} \, dt$$

$$- \frac{2}{F^2} \left(\frac{f_z}{|x'| + R'} + \frac{z'[- R' f_x \operatorname{sgn} x' - x' f_x + f - z' f_z]}{R'(|x'| + R')^2}\right) . \qquad (16)$$

The argument of the exponential integral in equation (16) generally takes moderate values, and accordingly, the exponential integral may be represented by the formula

$$E_1(Z_1) \approx e^{-Z_1} \left[\frac{\epsilon_1}{Z_1} + \sum_{j=2}^{18} \frac{\epsilon_j}{Z_1 + \delta_j}\right]. \qquad (17)$$

This approximation is due to Hershey [14], and clearly consists of approximating the exponential integral by a series of poles along the negative real axis. Numerical values for ϵ_j, $j = 1, \ldots, 18$ and δ_j, $j = 2, \ldots, 18$ are given in [14].

Using equation (17), the first term on the right-hand side of equation (16) may be expressed in the form

$$-\frac{\varepsilon_1}{\pi F^2}\int_{-\pi/2}^{3\pi/2}\frac{T_1 f_x \operatorname{sgn} x' - |x'|\,T_2}{T_1^2 + x'^2}\cos\theta\,d\theta \qquad (18)$$

$$-\frac{1}{\pi F^2}\sum_{j=2}^{18}\varepsilon_j\int_{-\pi/2}^{3\pi/2}\frac{(T_1\cos\theta + F^2\delta_j)\,f_x \operatorname{sgn} x' - |x'|\,T_2\cos\theta}{(T_1\cos\theta + F^2\delta_j)^2 + x'^2\cos^2\theta}\cos^2\theta\,d\theta$$

where

$$T_1 = -f\sin\theta + z'\cos\theta$$

$$T_2 = \sin\theta + f_z\cos\theta\ .$$

The first term in (18) may be evaluated immediately, using the calculus of residues, and becomes

$$-\frac{2\varepsilon_1}{F^2}\left[\frac{f_x z' \operatorname{sgn} x'}{R'(|x'| + R')} - \frac{z' f + f_z(x'^2 + f^2 + |x'|\,R')}{R'(|x'| + R')^2}\right].$$

The second term could likewise be evaluated by the calculus of residues, by summing appropriate contributions from each of the poles of the integrand, at which the denominator vanishes. In this case, however, the denominator is a quartic, and the formulas for its zeros are too complicated for practical use.

If the ship is very thin, f is a small quantity and terms involving it in the denominator of the second integral in (18) may be neglected. Consequently the denominator becomes approximately

$$z'\cos^4\theta + (2F^2\delta_j z' + x'^2)\cos^2\theta + F^4\delta_j^2,$$

which may easily be factorized. An approximation to the second integral in (18) may now be derived in a straightforward manner using the calculus of residues.

After much calculation, an approximation to equation (16) may be obtained in the form

$$(1 + f_x^2 + f_z^2)^{1/2} \frac{\partial N_1}{\partial N}$$

$$\approx -\frac{2\varepsilon_1}{F^2}\left[\frac{f_x z' \operatorname{sgn} x'}{R'(|x'| + R')} - \frac{z'f + f_z(x'^2 + f^2 + |x'|R')}{R'(|x'| + R')^2}\right]$$

$$-\frac{2}{F^2}\sum_{j=2}^{18}\varepsilon_j\left[\frac{(z'f_x \operatorname{sgn} x' - |x'| f_z)(1 - I_1)}{z'^2} + \frac{I_2 F^2 \delta_j f_x \operatorname{sgn} x'}{|x'|}\right]$$

$$-\frac{2}{F^2}\left(\frac{f_z}{|x'| + R'} + \frac{z'[-R' f_x \operatorname{sgn} x' - x'f_x + f - z'f_z]}{R'(|x'| + R')^2}\right) \qquad (19a)$$

where

$$I_1 = \frac{1}{4|x'|B}\left[\frac{U^3}{(U^2 + 4z'^2)^{1/2}} - \frac{V^3}{(V^2 + 4z'^2)^{1/2}}\right]$$

$$I_2 = \frac{1}{B}\left[\frac{U}{(U^2 + 4z'^2)^{1/2}} - \frac{V}{(V^2 + 4z'^2)^{1/2}}\right] \qquad (19b)$$

and

$$U = |x'| + B$$

$$V = |x'| - B \qquad (19c)$$

and

$$B = (x'^2 + 4F^2\delta_j z')^{1/2}. \tag{19d}$$

If the argument of the radical in equation (19d) is negative, B becomes imaginary and U and V in equation (19c) become complex conjugates. In this case, I_1 and I_2 in equation (19b) remain real, and (19a) continues to yield a real result. In addition, a meaningful limiting result may be derived for $B \to 0$.

The normal derivative of the wave-like term W_1 in equation (15) is obtained by straightforward differentiation. The resulting integral may be evaluated by the Hermite polynomial method (see [9], page 890, formula 25.4.46).

5. Approximate Green function for $y' \neq 0$

In this section an approximation will be derived, based on the second of Noblesse's [4] forms of the Havelock Green function. This may be written

$$G = -\frac{1}{R} + \frac{1}{R'} + N_2 + W_2,$$

where

$$N_2 = \frac{2}{\pi F^2} \int_{-\infty}^{\infty} \mathrm{Im}\{e^{Z_2} E_1(Z_2)\} \frac{t\,dt}{(1+t^2)^{1/2}}$$

and

$$W_2 = -\frac{4}{F^2} \int_0^{\infty} \exp\left[\frac{(t^2+1)\,z'}{F^2}\right] \sin\left[\frac{(t^2+1)^{1/2}}{F^2}(x' - t|y'|)\right] dt$$

$$- \frac{4}{F^2} \int_0^1 \exp\left[\frac{(1 - t^2)\, z'}{F^2} - \frac{t\,(1 - t^2)^{1/2}\,|y'|}{F^2}\right] \cos\left[\frac{(1 - t^2)^{1/2} x'}{F^2}\right] dt$$

and

$$Z_2 = \frac{t}{F^2}\left[i|y'|\,(1 + t^2)^{1/2} + x' - tz'\right]. \tag{20}$$

This form is clearly of little use in the direct evaluation of the Havelock Green function, since the remarks of section 3 apply here also. In addition, the wave-like part of the Green function, W_2, now contains two terms instead of one.

An approximate form of the Havelock Green function, based on equation (20), may be derived as in section 4 by substituting the approximate form (17) of the exponential integral into the expression for N_2. This yields

$$N_2 \approx - \frac{2\varepsilon_1|y'|}{\pi} \int_{-\infty}^{\infty} \frac{dt}{y'^2\,(1 + t^2) + (x' - z't)^2}$$

$$- \frac{2}{\pi} \sum_{j=2}^{18} \varepsilon_j|y'| \int_{-\infty}^{\infty} \frac{t^2\, dt}{y'^2 t^2\,(1 + t^2) + (x't - z't^2 + F^2\delta_j)^2}\,. \tag{21}$$

The first integral in this expression may be evaluated easily by the calculus of residues and becomes simply $\pi/(|y'|R')$. The second integral could likewise be evaluated using residue theory, but this would involve the determination of the roots of the denominator, which is a quartic in t. This is too complicated for practical use. Instead, the term $t^2\,(1 + t^2)$ in the denominator of the second integral in equation (21) is replaced by t^4, which is exact for $t = 0$ and accurate for large t. Then

$$N_2 \approx -\frac{2\varepsilon_1}{R'} + \frac{2}{\pi} \sum_{j=2}^{18} \varepsilon_j \, \mathrm{Im} \int_{-\infty}^{\infty} \frac{dt}{i|y'|\, t^2 + x't - z't^2 + F^2 \delta_j}.$$

The integral in this expression may now easily be evaluated using residue theory, and thus N_2 is given approximately by

$$N_2 \approx -\frac{2\varepsilon_1}{R'} - 4 \sum_{j=2}^{18} \varepsilon_j \left[\frac{1/2\,(T_1 + \{T_1^2 + T_2^2\}^{1/2})}{T_1^2 + T_2^2}\right]^{1/2}$$

with

$$T_1 = x'^2 + 4F^2 \delta_j z'$$

$$T_2 = 4F^2 \delta_j |y'|. \qquad (22)$$

The wave-making term W_2 in equation (20) may be evaluated using the Hermite polynomial method to perform the first integration, as in section 4. The second integral is evaluated using a Gaussian quadrature formula.

6. Conclusion

The application of the Havelock Green function to linearized ship-hydrodynamic problems typically results in an integral equation to be satisfied on the ship hull, with the normal derivative of the Havelock function as the kernel. In this case, the "near field" double-integral term may be expected to be of the same order of magnitude as the single-integral term responsible for the generation of waves far downstream, and cannot meaningfully be ignored in the evaluation of the kernel. However, the direct numerical evaluation of this double-integral term for use in integral

equations appears to require an unreasonably large amount of time on most present-day computers [6].

In order to avoid the direct numerical evaluation of this double-integral term, a number of closed-form approximations to it have been developed in this paper. Work is currently in progress to utilize them to determine a centerplane source distribution valid for the description of flow about a Wigley hull form at nonzero Froude number, by an iteration process similar to that used by Miloh and Landweber [5].

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